

International momentum strategies: a stochastic dominance approach

Wai Mun Fong^{a,*}, Wing Keung Wong^b, Hooi Hooi Lean^b

^a*Department of Finance and Accounting, National University of Singapore, 1 Business Link,
Singapore 119260, Singapore*

^b*Department of Economics, National University of Singapore, 1 Business Link, Singapore*

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Abstract

This paper applies recent econometric tests of stochastic dominance to examine an enduring puzzle in finance: the momentum effect in stock returns (J. Finance 48 (1993) 65). We use stochastic dominance tests to distinguish between the hypothesis that there exists general asset pricing models that can explain momentum versus the alternative hypothesis that there are no asset pricing models consistent with risk-averse investors that can rationalize that effect. Using stock index data for 24 countries over the period 1989–2001, we show that winner portfolios stochastically dominate loser portfolios at second and third order. These results are robust to two subperiods with different risk and return characteristics and survive reasonable transaction costs for international index funds. Our results indicate that the search for rational asset pricing explanations for the momentum effect may be a futile one.

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*Corresponding author. Tel.: +65-6874-6693; fax: +65-6779-2083.

E-mail address: bizfwm@nus.edu.sg (W.M. Fong).

1. Introduction

This paper applies recent econometric tests of stochastic dominance (Hadar and Russell, 1969; Hanoch and Levy, 1969; Rothschild and Stiglitz, 1970; Whitmore, 1970) to momentum strategies implemented on international stock market indices. The momentum effect, first documented by Jegadeesh and Titman (1993) refers to the tendency for portfolios of stocks that performed well (poorly) in the past 3–12 months to continue earning positive (negative) returns over the next 12 months. This pattern appears to be an anomaly from the perspective of efficient markets as empirical studies show that the effect cannot be explained away using standard asset pricing models such as the CAPM and Fama–French three-factor models (Jegadeesh and Titman, 1993; Grundy and Martin, 2001). Nonetheless, the search for more general asset pricing models continues on the premise that existing models may be inadequate because of omitted risk factors. This paper questions the utility of this exercise.

We use stochastic dominance theory to distinguish between the hypothesis that there exists some (more general) asset pricing models that can explain the momentum effect versus the alternative hypothesis that there are no asset pricing models consistent with risk-averse investors that can rationalize that effect. If winner portfolios stochastically dominate loser portfolios at second or higher orders, then it is unlikely that the problem is due to omitted risk factors but more likely that momentum reflects market inefficiency.

The primary advantage of the stochastic dominance approach is that it provides a very general framework to assess portfolio choice without the need for asset pricing benchmarks. In addition, stochastic dominance theory makes no assumptions about the distribution of asset returns and minimal assumptions about investors' utility functions.

We apply recently developed tests of stochastic dominance by Davidson and Duclos (2000) and Barrett and Donald (2003) to momentum strategies implemented on 24 international stock indices for the period 1989 to 2000. Previous studies of international momentum strategies, e.g., Chan et al. (2000), report significant risk-adjusted profits for different sample periods. Our stochastic dominance approach yields consistent but more general results. Specifically, we find that winners dominate losers by second- and third-order stochastic dominance, implying that *all* investors with strictly concave utility functions would have preferred to buy winners and sell losers over the sample period. Our results are robust to two subperiods and survive realistic transaction costs for index funds. Overall, the evidence is discouraging for efficient market advocates, for it implies that asset pricing models based on the assumption that investors are risk averse will not explain the momentum effect. In other words, the failure of existing asset pricing models to explain momentum may have less to do with omitted risk factors than their ability to capture the irrational aspects of momentum investors (e.g., Daniel et al., 1998; Hong and Stein, 1999).

This rest of this paper is organized as follows. Section 2 summarizes the empirical evidence on the momentum effect. An overview of stochastic dominance theory is

presented in Section 3. Section 4 describes the data and the momentum strategy used in this study. Preliminary results on the profitability of the momentum strategy are reported in Section 5. Section 6 introduces some recent econometric tests of stochastic dominance. Results of stochastic dominance tests for momentum portfolios are reported in Section 7, followed by some robustness tests in Section 8. Section 9 concludes with a summary of the main findings.

2. Evidence on the profitability of momentum strategies

The momentum effect, first discovered by Jegadeesh and Titman (1993), is an enduring puzzle in finance. Jegadeesh and Titman (1993) show that a simple strategy of buying stocks with high returns over 3 to 12 months and selling stocks with low returns over the same period produces annualized returns of about 12% for the following year. The momentum effect also appears in many non-U.S. markets. Rouwenhorst (1997) finds that momentum strategies are profitable for stocks in 12 European markets while Griffin et al. (2003) document significant momentum profits for a dataset of 40 countries. Chan et al. (2000) draw similar conclusions for momentum strategies implemented on international stock market indices.

Much research has focused on whether momentum profits can be explained by risk. Jegadeesh and Titman (1993) find that momentum cannot be explained by exposure to market risk alone. Fama and French (1996) propose two other risk measures. The first is a high minus low book-to-value (HML) risk factor, which is usually interpreted as a firm distress proxy. The second is a small minus large firm size (SML) risk factor to proxy for the higher risk and lower liquidity of small firms. Fama and French (1996) find that their unconditional three-factor model cannot account for momentum profits. Grundy and Martin (2001) show that neither can a conditional three-factor model explain momentum. Conrad and Kaul (1998) conjecture that the three-factor model may be misspecified and that momentum returns may reflect an omitted component of returns that differs across stocks. In other words, momentum profits may simply be due to cross-sectional dispersions in unconditional expected returns. Grundy and Martin (2001) test the Conrad–Kaul conjecture by using each stock as its own risk control. They find that the momentum strategy still yields excess returns of 9.24% per annum in the period 1966–1995. Jegadeesh and Titman (2002) also show that differences in unconditional expected returns cannot explain momentum profits.

Moskowitz and Grinblatt (1999) argue that momentum profits may also be due to industry risk exposures that are not captured by standard factor models. Grundy and Martin (2001) test for industry risk but find that it cannot fully explain the momentum effect. In particular, a random industry strategy still earns statistically significant returns in months other than January.

Business cycle risk is another potential explanation of the momentum effect. If the expected return to the momentum strategy is related to business cycle risk, then one would expect momentum profits to be lower in poor economic states than in good economic states. Griffin et al. (2003) provide international evidence against this

hypothesis: momentum profits appear to be large and statistically significant across good and bad economic states. They also show that the multifactor macroeconomic model of Chordia and Shivakumar (2002) cannot explain momentum across markets.

In short, current risk-based explanations fail to account fully for the momentum effect. As Grundy and Martin (2001) summarize succinctly, “A full understanding of the source of the risk-adjusted profitability of the momentum strategy remains an open question.”

The failure of risk-based models has led to research which attempts to explain momentum in terms of behavioral inconsistencies of investors. For example, Daniel et al. (1998) attribute momentum to the fact that investors are overconfident and tend to overreact to confirming news while attributing wrong forecasts to external noise. An increase in overconfidence following the arrival of confirming news leads to further overreaction, generating momentum in stock prices. Hong and Stein (1999) also attribute momentum to overreaction, and add that overreaction is a delayed response to the slow diffusion of private information. They also conjecture that the less risk averse investors are, the greater is the degree of overreaction. This implies that momentum should be stronger in good economic states than in bad economic states.

There is increasing evidence supporting some aspects of behavioral explanations of momentum. For example, behavioral models imply that momentum profits should be better explained in terms of stock-specific risks than systematic risks. Consistent with this hypothesis, Grundy and Martin (2001) show that a momentum strategy that defines winners and losers based on stock-specific returns (residuals from the three-factor model) is significantly more profitable than a strategy that ranks stocks on total returns. Other studies focus on more specific aspects of idiosyncratic risk. For example, Chan et al. (1996) show that momentum profits coexist with earnings momentum, while Lee and Swaminathan (2000) document stronger momentum effects in stocks with high turnover.

Cooper et al. (2004) test the Hong–Stein hypothesis by conditioning momentum profits on the state of the market. They find that momentum profits arise only following up-market periods. They interpret this finding as consistent with the Hong–Stein hypothesis that there is more delayed overreaction and hence stronger momentum following periods when the market is up and investors are less risk averse.

Although the evidence points to investor irrationality as the primary cause of momentum, many aspects of behavioral theories are intrinsically hard to test. If momentum is due to overreaction, momentum returns must eventually reverse as investors correct the mispricing. Unfortunately, this hypothesis is difficult to refute since behavioral theories do not specify a precise time frame for such reversals to occur. Thus, to behavioral skeptics, the issue of whether momentum profits are compensation for risk or reflect investor irrationality remains an open question.

The goal of this paper is to shed light on this debate by asking whether the momentum effect is consistent with *any* rational asset pricing model. To address this

issue, a general testing framework that depends only on primitive assumptions is needed. Stochastic dominance theory is useful in this regard as it provides rules for unambiguous ranking of risky choices based on utility theory rather than narrowly specified asset pricing models.

3. Stochastic dominance

Stochastic dominance theory provides a general framework for studying economic behavior under uncertainty. Hadar and Russell (1969), Hanoch and Levy (1969), Rothschild and Stiglitz (1970) and Whitmore (1970) laid the utility foundations of stochastic dominance analysis. It is well known that stochastic dominance rules are more general than standard mean variance analysis which is valid only if utility functions are quadratic or if asset returns conform to a normal distribution.¹ Stochastic dominance rules are also encompassing for any well defined von Neumann–Morgenstern set of utility functions. For example, stochastic dominance rules which apply to the general class of non-decreasing, concave utility functions will give consistent rankings for *all* members of this class (Russell and Seo, 1978, 1989). This subset includes interesting types such as the downside risk or mean-semivariance form of utility functions discussed by Markowitz (1952, 1959).

Fishburn (1989) shows that stochastic dominance rules extend to more nonlinear utility functions based on substantially weaker axioms than von Neumann–Morgenstern linear utility theory.

Levy (1998) provides an up-to-date summary of stochastic dominance and its applications in economics and finance. In finance, the stochastic dominance approach has been used to study option pricing (Levy, 1985), the small-firm effect (Seyhun, 1993) and portfolio selection (Post, 2003). We believe ours is the first paper that uses stochastic dominance theory to study the momentum effect.

The most common rules used in the stochastic dominance literature are those relating to first, second and third order stochastic dominance. Let F and G be the cumulative distributions of two risky assets, x the uncertain return and U , a von-Neumann–Morgenstern utility function. Assume all investors are non-satiated, i.e., $U'(x) \geq 0$. Then, F first-order dominates G for all non-satiated investors if

$$F(x) \leq G(x) \quad \text{for all } x. \quad (1)$$

Clearly, first-order stochastic dominance (FSD) is a highly stringent condition, which limits its applicability in actual choice situations.

We focus on second and third-order stochastic dominance as these concepts have compelling utility interpretations in terms in investor risk aversion and skewness preference respectively. F dominates G at the second order for all (risk averse)

¹Consider the returns for two risky assets, X and Y . Suppose $E(X) = E(Y)$ and $\sigma_X^2 > \sigma_Y^2$. There will exist some increasing and strictly concave utility functions such that $EU(X) \geq EU(Y)$. If the means are unequal, no relation involving standard deviations is known to be necessary for a risk averse individual to prefer X over Y .

investors with utility functions $U'(x) \geq 0$ and $U''(x) \leq 0$ if and only if

$$\int_{-\infty}^{\infty} [G(z) - F(z)] dz \geq 0 \quad \text{for all } x. \quad (2)$$

Third-order stochastic dominance adds the assumption that individuals prefer return distributions which are more positively skewed. Specifically, F dominates G at the third order for all risk averse investors with $U'(x) \geq 0$, $U''(x) \leq 0$ and $U'''(x) \geq 0$ if and only if $\mu_F > \mu_G$ where μ denotes expected return and

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [G(z) - F(z)] dz dv \geq 0 \quad \text{for all } x. \quad (3)$$

4. Data and methodology

4.1. Data

Following Chan et al. (2000), we implement momentum strategies on international stock market indices. Our data covers 24 indices compiled by Morgan Stanley Capital International (MSCI). Every MSCI index comprises large capitalization and actively traded stocks, thus mitigating biases due to non-synchronous trading or bid-ask spreads. The sample period is from January 1989 through December 2001. All data are sourced from Datastream.

The 24 countries in our sample are Australia, Austria, Belgium, Canada, Denmark, France, Germany, Hong Kong, Indonesia, Italy, Japan, Korea, Malaysia, Netherlands, Norway, Singapore, South Africa, Spain, Sweden, Switzerland, Taiwan, Thailand, UK and US. This sample includes all the well established markets as well as a few emerging markets.

4.2. Momentum strategy

We implement a momentum strategy by buying stock indices with high returns over the previous 1 to 12 months and selling stock indices with low returns over the same period. Specifically, on each day, t , we rank all 24 markets based on their compounded returns from $t-J$ to $t-1$, where $J = 22, 66, 132, 198$ and 264 days (approximately 1, 3, 6, 9 and 12 months, respectively).² This is known as the ranking period. We then form a winner (W) portfolio comprising the four markets with the highest ranking period returns and a loser (L) portfolio comprising the four markets with the lowest ranking period returns. Next, we skip one day and invest in a momentum portfolio on day $t+1$ that buys W and short sells L . We skip one day primarily to mitigate microstructure biases due to bid-ask bounce effects. The

²The exact average number of days is 22, 65, 130, 195 and 260.

momentum portfolio is held for K days (the holding period) and its geometric mean daily return is computed assuming equal weights.³ To reduce the number of portfolio combinations, we let K be the same as J . For example, if the ranking period is 22 days, the momentum portfolio is held for 22 days.

5. Profitability of momentum strategy

Table 1 reports ranking period returns of winner, loser and momentum ($W-L$) portfolios for the overall sample period. All returns are in local currency. Therefore, we take the perspective of a multinational investor.⁴

For all ranking periods, winner returns are positive while loser returns are negative. The mean daily return across all horizons is 0.216% for winners and -0.174% for losers, which translates to annualized returns of 54% and -43.5% , respectively.

Clearly, this was a period with very strong momentum for both extreme portfolios, with consequently very high returns for the momentum portfolio. For example, the 1-month mean return to the momentum portfolio is 0.718% per day (180% per annum) while the 12-month mean return is 0.229% per day (57.3% per annum). The distribution of momentum portfolio returns is right-skewed. Together, these results hint of potentially large profits from a strategy of buying past winners and shorting past losers. Table 2 reports holding period returns of such a strategy.

Results for the full sample period are shown in Panel A. In contrast to the ranking period, winners show continuation in returns but losers experience reversals. Thus, a strategy of buying winners without short selling losers earned larger profits than a more costly strategy that requires short selling losers. Nonetheless, based on the t -test, the momentum strategy is profitable across all holding periods. The 1-month and 6-month momentum portfolios produced the highest average daily return of 0.036% and 0.035%, respectively, while the 12-month portfolio has the lowest mean daily return (0.0053%).

The t -test assumes that returns are i.i.d. normal. To check whether our results are affected by violations of this assumption, we performed a bootstrap test whereby we scramble the stock market indices simultaneously to remove any dependence structure for each market while preserving cross-market correlations. We then execute our momentum strategy on 5,000 bootstrap samples and compute the empirical p -values, i.e., the percentage of the simulations generating higher mean returns than the actual sample. If the p -value is above 5%, we infer that momentum profits detected by the t -test are due to chance or are biased by non-normality. The

³To minimize transaction costs, we assume that there is no rebalancing within the holding period. Section 8 discusses transaction costs in greater detail.

⁴To check whether our results are sensitive to currency fluctuations, we also compute returns in U.S. dollars (i.e., taking the perspective of a U.S. investor). The results (not reported) show that average U.S. dollar momentum profits are larger than local currency profits for all holding periods. Thus, our results are robust to U.S. dollar conversion. For the rest of this paper, we will only discuss results based on local currencies.

Table 1

Ranking period average returns of winner, loser and momentum portfolios

This table reports ranking period average returns (in percentage) of winner (W), loser (L) and momentum (long winner and short loser) portfolios. The sample period is January 1, 1989 to December 31, 2001 (3391 observations). At the beginning of each day, the geometric mean returns of 24 stock market indices are computed over a J -day ranking period where $J = 22, 66, 132, 198$ and 264 days (approximately 1, 3, 6, 9, and 12 months). $W(L)$ comprises the portfolio with the four strongest (weakest) markets in the ranking period. All portfolios are equally weighted. The reported average returns are the arithmetic means of all geometric average returns over overlapping periods. Numbers in parentheses are heteroskedasticity-and-autocorrelation consistent t statistics computed using the method of Gallant (1987).

Ranking period returns	Winner (W)	Loser (L)	$W-L$
1-month	0.378	−0.340	0.718
t -stat	(29.49)	(−26.99)	(65.37)
Skewness	0.83	−1.14	1.12
3-month	0.236	−0.199	0.435
t -stat	(28.96)	(−23.53)	(62.99)
Skewness	0.74	−1.06	1.00
6-month	0.176	−0.136	0.312
t -stat	(30.64)	(−23.10)	(66.36)
Skewness	0.23	−0.98	0.82
9-month	0.152	−0.104	0.256
t -stat	(32.12)	(−22.50)	(63.54)
Skewness	0.64	−0.46	1.07
12-month	0.140	−0.089	0.229
t -stat	(33.32)	(−21.25)	(62.15)
Skewness	0.57	−0.47	1.18

bootstrap results (not reported) show that the p -values are uniformly below 5% across all time horizons. We conclude that the documented momentum profits are not spurious.

To test whether momentum profits for our sample persisted through time, we implement our momentum strategy over two subperiods with different return and volatility characteristics. The first subperiod is from January 1989 through December 1996. This is a bullish period for most stock markets. Of the 24 stock markets in the sample, 22 registered positive holding period returns. The mean geometric average return across all the markets is 0.024% per day. The second subperiod, from January 1997 through December 2001, is relatively more bearish and volatile than the first subperiod. Major events which contributed to this volatility include the Thai baht devaluation (July 1997) which adversely affected many other South East Asian stock markets, the Russian debt crisis (August 1998) and the bursting of the Internet stock bubble (March 2000). Only 16 markets registered positive holding period returns in this period. The mean geometric average return for all the markets was −0.031%.

Panel B reports momentum results for the first subperiod. Both winners and losers have positive returns, indicating momentum for winners but reversals for losers.

Table 2

Holding period average returns of winner, loser and momentum portfolios

This table reports holding period average returns (in percentage) of winner (*W*), loser (*L*) and momentum (long winner and short loser) portfolios. The full sample period is January 1, 1989 to December 31, 2001 (3391 observations). Subperiod 1 is from January 1, 1989 through December 31, 1996 (2087 observations) and subperiod 2 is from January 1, 1997 through December 31, 2001 (1304 observations).

At the beginning of each day, the geometric mean returns of 24 stock market indices are computed over a *J*-day ranking period where *J* = 22, 66, 132, 198 and 264 days (approximately 1, 3, 6, 9, and 12 months). After skipping a day, a momentum portfolio is formed by longing a winner portfolio (*W*) comprising the four strongest markets in the ranking period and shorting a loser portfolio (*L*) comprising the four weakest markets in the ranking period. The momentum portfolio is then held for *K* days where *K* is the same number of days as in the ranking period. The table reports the average holding period mean return of the 5 momentum portfolios. This is computed as the arithmetic mean of all geometric mean returns over overlapping periods. Numbers in parentheses are heteroskedasticity-and-autocorrelation consistent *t* statistics computed using the method of Gallant (1987). Bold entries denote returns that are significantly different from zero at 10% or lower.

Holding period	W	L	$W-L$		
			All months	January	Non-January
<i>A. Full sample</i>					
1-month	0.0505 (4.12)	0.0148 (1.11)	0.0357 (3.30)	−0.0391 (−1.22)	0.0426 (3.83)
3-month	0.0332 (4.01)	0.0292 (3.04)	0.0040 (0.49)	−0.0051 (−0.30)	0.0049 (0.56)
6-month	0.0521 (9.09)	0.0173 (2.58)	0.0348 (4.91)	0.0344 (2.20)	0.0349 (4.66)
9-month	0.0365 (8.04)	0.0124 (2.18)	0.0241 (3.57)	0.0225 (1.62)	0.0243 (3.38)
12-month	0.0239 (5.99)	0.0187 (3.65)	0.0053 (0.84)	−0.0104 (−0.67)	0.0067 (1.02)
<i>B. Subperiod 1</i>					
1-month	0.0511 (4.09)	0.0272 (2.07)	0.0239 (2.08)	−0.0235 (−0.64)	0.0283 (2.41)
3-month	0.0274 (3.17)	0.0387 (4.17)	−0.0113 (−1.36)	−0.0230 (−1.22)	−0.0102 (−1.16)
6-month	0.0453 (6.63)	0.0326 (4.89)	0.0127 (1.69)	−0.0138 (−0.82)	0.0151 (1.93)
9-month	0.0406 (7.65)	0.0214 (4.08)	0.0193 (3.01)	0.0112 (0.94)	0.0200 (2.93)
12-month	0.0341 (7.27)	0.0261 (5.81)	0.0080 (1.37)	−0.0228 (−1.57)	0.0109 (1.79)
<i>C. Subperiod 2</i>					
1-month	0.0496 (2.17)	−0.0050 (−0.19)	0.0545 (2.79)	−0.0643 (−1.25)	0.0655 (3.40)
3-month	0.0425 (2.85)	0.0139 (0.76)	0.0286 (1.91)	0.0240 (0.87)	0.0290 (1.92)
6-month	0.0631 (7.16)	−0.0072 (−0.59)	0.0703 (5.78)	0.1122 (6.70)	0.0664 (5.43)
9-month	0.0299 (4.17)	−0.0019 (−0.17)	0.0319 (2.45)	0.0409 (1.50)	0.0310 (2.40)
12-month	0.0076 (1.25)	0.0067 (0.66)	0.0009 (0.08)	0.0095 (0.33)	0.0001 (0.01)

Despite this, the momentum strategy of longing winners and shorting losers is generally profitable (the 3 and 12-month holding periods being the exceptions). The 1- and 9-month momentum portfolios generated the highest mean daily return of 0.0239% and 0.0193%, respectively.

Results for the second subperiod are shown in Panel C. The mean return for winners across all holding periods is 0.039%, which is only slightly below their mean return in the first subperiod (0.04%). Therefore, winner momentum has clearly persisted over the two periods. On average, returns for losers are significantly smaller than in the first subperiod, and are in fact negative for the 1, 6 and 9-month holding periods. Consequently, the momentum strategy is more profitable in the second subperiod than in the first. Across all horizons, the mean daily return of the momentum portfolio is 0.0372%, with the 1- and 6-months strategy performing best.

In summary, the momentum strategy has been highly profitable over the entire sample period. This profit is mainly due to winners, which showed consistently strong returns continuation. Could this persistence in returns be explained by macroeconomic risks? If investing in past winners is a risky strategy, we would expect returns to winners to be positive when the market is bullish but negative when the market is bearish. The data does not support this hypothesis. If anything, winner profits were higher in the more bearish second subperiod, suggesting that the profits cannot be explained in terms of economy-wide systematic risk. Our results are consistent with those of Griffin et al. (2003) who find that momentum profits are positive in *all* economic states, regardless of whether economic states are defined by world GDP growth rate or aggregate stock market returns.

6. Stochastic dominance tests

Several methods for testing stochastic dominance have been used in the econometrics literature. An early test for stochastic dominance was proposed by McFadden (1989). This is a Kolmogorov–Smirnov type test for first-order stochastic dominance for independent samples with equal number of observations. The asymptotic distribution of the test statistic for $s \geq 2$ is analytically intractable. Barrett and Donald (2003) develop a Kolmogorov–Smirnov type test for stochastic dominance of any pre-specified order. The test applies to two independent samples with possibly unequal sample sizes. Hereafter, this test will be referred to as the KS test.

Davidson and Duclos (hereafter DD, 2000) propose a test that is more general and computationally simpler than the KS test. The DD test simplifies by comparing cumulative distribution functions over an arbitrary grid of points. An advantage of this test is that it can be applied to dependent samples. Since the KS and DD tests have advantages as well as disadvantages, we use both tests to assess the statistical significance of momentum profits. Evidence that both tests produce similar results would give us a greater degree of confidence in our results. As these tests are relatively new, we provide a brief description of the tests as well as the null and alternative hypotheses in the next two sections.

6.1. The Barrett–Donald KS test

Let $\{W_i\}$, $i = 1, 2, \dots, N$ be an i.i.d. sample of returns to winners from a population with cumulative distribution function (CDF), $F_W(x)$. Without loss of generality, assume that all CDFs have common support $[0, \bar{x}]$ where $\bar{x} > 0$ and are continuous in $[0, \bar{x}]$. Define $D_W^s(x)$ as the function that integrates F_W to order $s-1$. That is,

$$D_W^1(x) = F_W(x),$$

$$D_W^2(x) = \int_0^x F_W(u) du = \int_0^x D_W^1(u) du,$$

$$D_W^3(x) = \int_0^x \int_0^y F_W(v) dv du = \int_0^x D_W^2(u) du.$$

Let $\{L_i\}$, $i = 1, 2, \dots, N$ be an i.i.d. sample of returns to losers, with cumulative distribution function, $F_L(x)$. Define $D_L^s(x)$ analogously.

The KS test evaluates the following null and alternative hypotheses:

$$H_0^s : D_W^s(x) \leq D_L^s(x) \quad \forall x \in [0, \bar{x}],$$

$$H_1^s : D_W^s(x) > D_L^s(x) \quad \text{for some } x \in [0, \bar{x}].$$

The null hypothesis is that winners dominate losers while the converse is implied by the alternative hypothesis. The following test statistic proposed by [Barrett and Donald \(2003\)](#) can be used to test H_0^s :

$$\bar{K}_s = \left(\frac{N^2}{2N} \right)^{1/2} \sup_x [\bar{D}_W^s(x) - \bar{D}_L^s(x)]. \quad (4)$$

For $s \geq 2$, it is analytically intractable to derive critical values of the test statistic because the limiting distribution of $\bar{K}_s$ depends on the underlying CDFs. Following [Barrett and Donald \(2003\)](#), we simulate p -values using an arbitrarily fine grid to calculate the suprema of $\bar{K}_s$.⁵

6.2. The Davidson–Duclous (DD) test

[Davidson and Duclous \(2000\)](#) propose a test of stochastic dominance that applies to both independent samples as well as dependent samples drawn from a joint distribution, e.g., the returns of two funds over the same period. They define the following hypotheses:

1. $H_0 : D_W^s(x_k) = D_L^s(x_k) \quad \forall x_k, k = 1, \dots, K$
2. $H_A : D_W^s(x_k) \neq D_L^s(x_k) \quad \text{for some } x_k$
3. $H_{A1} : D_W^s(x_k) > D_L^s(x_k)$
4. $H_{A2} : D_W^s(x_k) < D_L^s(x_k).$

⁵For details, see [Barrett and Donald \(2003, p. 79\)](#).

Thus, alternate hypothesis A1 implies that losers dominate winners while alternate hypothesis A2 implies the reverse.

The following the t -statistic can be used to test the null hypothesis:

$$T^s(x) = \frac{\hat{D}_W^s(x) - \hat{D}_L^s(x)}{\sqrt{\hat{V}^s(x)}}, \quad (5)$$

where $V^s(x)$ is the variance of the integrals of the cumulative distribution functions. Under the null hypothesis, $T^s(x)$ is asymptotically distributed as a standard normal variate.

Unlike the KS test, the DD test is implemented over a grid of pre-selected points, $x_k, k = 1, \dots, K$ and the null hypothesis is rejected only if the *largest* t -statistic across these grid points is significant. Our choice of K is guided by the results of various simulation studies. Barrett and Donald (2003) and Tse and Zhang (2004) show that for reasonably large samples (> 500 observations), the DD test works well for $K = 10$. Actual applications may require a finer grid because as Barrett and Donald (2003, p. 91) point out, a coarse grid may miss important differences in the distributions. To mitigate this, we partition the data equally by using 10 major grids. Each major interval is in turn partitioned into 10 equal sub-intervals. The DD statistic is then calculated using the 10 grids within each sub-interval. To control for joint test size, statistical inference is based on the student maximum modulus (SMM) distribution for $K = 10$ and infinite degrees of freedom. The 5% asymptotic critical value of the SMM distribution is 3.254 from Stoline and Ury (1979).

7. Results of stochastic dominance tests

Results of the KS test are shown in Table 3. This table reports p -values of the KS test for second order stochastic dominance (H_2^0) and third-order stochastic dominance (H_3^0) respectively. Under H_2^0 , the column “ $W \succ_2 L$ ” shows p -values for testing the null hypothesis that winners weakly dominate losers at second order, while the column “ $L \succ_2 W$ ” tests the opposite hypothesis. P -values under H_3^0 are interpreted analogously. All p -values are computed using simulations based on the procedure in Barrett and Donald (2003).

Results for the full sample period (Panel A) show that p -values for $W \succ_2 L$ and $W \succ_3 L$ are well above 5% while p -values for the opposite hypotheses are nearly zero across all holding periods. These results provide strong evidence of winner dominance over the entire sample period.

Subperiod results (Panels B and C) reveal that winner dominance is more pervasive in the second subperiod (1997–2001). The fact that the momentum strategy is more successful in the latter subperiod suggests that trend-chasing behavior in stock markets has not disappeared in recent times.

The DD test results are shown in Table 4. Recall that the DD test rejects the null hypothesis if none of the DD statistics are significantly positive and at least *some* of the DD statistics are significantly negative. Since the DD test does not specify what

Table 3

Results of KS Test for stochastic dominance

P-values are reported for the KS test of stochastic dominance (Barrett and Donald, 2003) for winner and loser portfolios. The full sample period is from January 1, 1989 to December 31, 2001 (3391 observations). Subperiod 1 is from January 1, 1989 through December 31, 1996 (2087 observations) and subperiod 2 is from January 1, 1997 through December 31, 2001 (1304 observations).

The null hypotheses H_2^0 and H_3^0 relate to second- and third-order stochastic dominance respectively. Under H_2^0 , the column $W \succ_2 L$ contains *p*-values for testing the hypothesis that the CDF of winners is less than or equal to the CDF of losers, while the column $L \succ_2 W$ contains *p*-values for testing the opposite hypothesis. A similar interpretation holds for H_3^0 . The distribution of the KS statistic is obtained by simulations using Brownian Bridge processes generated by i.i.d. $N(0,1)$ observations (see Barrett and Donald (2003, pp. 79–80)). The number of grid points for computing the suprema of the KS statistic was fixed at 100. Sample sizes used in the simulations correspond to the actual number of observations for each sample period. Bold entries indicate that the null hypothesis is accepted at 5% significance level.

	H_2^0		H_3^0	
	$W \succ_2 L$	$L \succ_2 W$	$W \succ_3 L$	$L \succ_3 W$
<i>A. Full sample</i>				
1-month	0.583	0.000	0.537	0.000
3-month	0.685	0.000	0.642	0.000
6-month	0.673	0.000	0.684	0.000
9-month	0.712	0.000	0.653	0.000
12-month	0.713	0.000	0.681	0.000
<i>B. Subperiod 1</i>				
1-month	0.580	0.000	0.021	0.000
3-month	0.001	0.581	0.003	0.655
6-month	0.039	0.000	0.147	0.000
9-month	0.068	0.00	0.259	0.000
12-month	0.232	0.00	0.394	0.000
<i>C. Subperiod 2</i>				
1-month	0.663	0.000	0.614	0.000
3-month	0.680	0.000	0.635	0.000
6-month	0.663	0.000	0.627	0.000
9-month	0.689	0.000	0.645	0.000
12-month	0.676	0.000	0.632	0.000

percentage of DD statistics must be significantly negative to reject the null hypothesis, to be conservative, we use a 50% cutoff point. That is, we infer that winners dominate losers if at least 50% of the DD statistics are significantly negative and no DD statistics are significantly positive.

Two main points may be noted from the full sample results (Panel A). First, in terms of second-order stochastic dominance, no DD statistic is significantly positive, while DD statistics for most holding periods are significantly negative (exception is the 3-month holding period). These results strongly indicate that all risk-averse investors would have preferred winners over the entire sample period. Second, evidence for winner dominance is generally stronger at third order than at second

Table 4

Results of DD test for stochastic dominance

Results of the [Davidson–Duclous \(2000\)](#) test of stochastic dominance are reported for winner-loser portfolios. The full sample period is from January 1, 1989 to December 31, 2001 (3391 observations). Subperiod 1 is from January 1, 1989 through December 31, 1996 (2087 observations) and subperiod 2 is from January 1, 1997 through December 31, 2001 (1304 observations).

DD test statistics are computed over a grid of 100 daily winner and loser portfolio returns. The table reports the percentage of DD statistics which are significantly negative or positive at the 5% significance level, based on the asymptotic critical value of 3.254 of the studentized maximum modulus (SMM) distribution. Bold entries indicate winners dominate losers at the 5% significance level based on a 50% cutoff point.

	SSD		TSD	
	%DD<0	%DD>0	%DD<0	%DD>0
<i>A. Full sample</i>				
1-month	73	0	66	0
3-month	50	0	81	0
6-month	85	0	80	0
9-month	95	0	94	0
12-month	76	0	95	0
<i>B. Subperiod 1</i>				
1-month	55	0	45	0
3-month	0	7	0	0
6-month	49	32	26	43
9-month	60	26	45	34
12-month	44	0	0	0
<i>C. Subperiod 2</i>				
1-month	83	0	78	0
3-month	88	0	85	0
6-month	85	0	81	0
9-month	95	0	94	0
12-month	63	0	95	0

order. This implies that investors who prefer more positive skewness would also have chosen to buy winners and sell losers. This preference may be due to the fact that the distribution of winner returns is right-skewed ([Table 1](#)).

Subperiod results are consistent with the KS test. In particular, there is only weak evidence of winner dominance in the first subperiod. This result is not entirely surprising. Because the first subperiod was a bullish period for most stock markets, even past losers produced positive returns for each holding period, thus reducing the profitability of the momentum strategy. On the other hand, there is clear evidence of winner dominance in the second subperiod, where at least 63% of DD statistics are significantly negative and none are significantly positive. This result is consistent with the KS test. The success of the momentum strategy in this latter period is due to the continued rise in the price of winners as well as the fall in the price of losers.

In summary, both tests provide strong evidence of the momentum effect in international stock markets. Although these results are based on second and third order stochastic dominance, separate tests using the more general concept of first order stochastic dominance also indicate winner dominance for the 6- and 9-month holding period. These results cast doubt on the argument that existing asset pricing models do not adequately explain momentum because of omitted risk factors. On the contrary, they suggest that the search for more general asset pricing models to explain momentum may be a futile exercise.

8. Robustness checks

8.1. Results based on five-day skip

The results reported in Section 7 were based on momentum portfolios formed by skipping one day following the end of the ranking period to avoid bid–ask bounce biases. It may be argued that one day is too short for bid–ask bounce effects to dissipate. Furthermore, a one-day skip may not be long enough to eliminate spurious momentum profits due to nonsynchronous trading of index component stocks. To address these concerns, we repeat all our tests using a one week (5-business day) skip period. This should mitigate the problem of nonsynchronous trading to the extent that index component stocks trade at least once a week. The results are shown in Table 5.

Except for the 1 and 12-month holding periods where average returns are significantly lower with a 5-day skip, results for the other holding periods are quite robust. For example, for the popular 6-month momentum strategy, the average daily return is 0.0352% with 5-day skip, which is very close to the 0.0348% return using 1-day skip (Table 2). The same holds for the 9-month momentum strategy (0.0237% versus 0.0241%). Thus, it is unlikely that all momentum profits can be explained by microstructure biases or nonsynchronous trading. The statistical significance of momentum profits is confirmed by the DD test, the results of which are very similar to those reported in Table 4 based on 1-day skip. As before, there is strong evidence that winners stochastically dominate losers over the full sample and second subperiods.⁶

8.2. Is momentum unique to emerging markets?

Harvey (1995) and Bekaert et al. (1997) show that emerging market stock returns are more autocorrelated than returns to developed markets. Low liquidity may be the main reason for the higher autocorrelation. If momentum profits are solely

⁶Another way to mitigate nonsynchronous trading bias is to perform tests using lower frequency, e.g., weekly data instead of daily data. Results of the DD test based on weekly returns with a one-week skip (unreported) show that winners still dominate losers at second and third order for the 6- and 9-month holding period.

Table 5

Profitability of momentum strategies implemented with 5-day skip

This table reports the profitability of momentum strategies in which momentum portfolios are formed by skipping 5 days after the end of each ranking period. The second column reports average holding period returns of long winner and short loser (*W-L*) momentum portfolios. Average holding period returns are computed as the arithmetic average of all geometric mean returns across overlapping periods. The full sample period is from January 1, 1989 to December 31, 2001 (3391 observations). Subperiod 1 is from January 1, 1989 through December 31, 1996 (2087 observations) and subperiod 2 is from January 1, 1997 through December 31, 2001 (1304 observations).

Results of the [Davidson–Duclos \(2000\)](#) test of stochastic dominance are shown in the last four columns. DD test statistics are computed over a grid of 100 daily winner and loser portfolio returns. The table reports the percentage of DD statistics which are significantly negative or positive at the 5% significance level, based on the asymptotic critical value of 3.254 of the studentized maximum modulus (SMM) distribution. Bold entries indicate winners dominate losers at the 5% significance level based on a 50% cutoff point.

Sample period	<i>W-L</i>	DD Test			
		SSD		TSD	
		%DD<0	%DD>0	%DD<0	%DD>0
<i>A. Full sample</i>					
1-month	0.0232 (2.14)	74	0	68	0
3-month	0.0018 (0.22)	41	0	64	0
6-month	0.0352 (4.93)	89	0	85	0
9-month	0.0237 (3.49)	97	0	96	0
12-month	0.0039 (0.62)	69	0	93	0
<i>B. Subperiod 1</i>					
1-month	0.0106 (0.91)	23	0	40	0
3-month	−0.0155 (−1.83)	0	30	0	8
6-month	0.0144 (1.90)	51	34	30	45
9-month	0.0197 (3.05)	61	27	46	36
12-month	0.0072 (1.22)	44	0	3	0
<i>C. Subperiod 2</i>					
1-month	0.0433 (2.23)	83	0	79	0
3-month	0.0295 (2.01)	85	0	81	0
6-month	0.0687 (5.59)	89	0	86	0
9-month	0.0302 (2.31)	97	0	96	0
12-month	−0.0014 (−0.11)	59	0	89	0

Table 6

Stochastic dominance in developed stock markets

This table reports performance statistics of the momentum strategy for 16 developed stock markets. The sample period is from January 1, 1989 to December 31, 2001 (3391 daily observations). The portfolio formation procedure is the same as that described in the notes to Table 1. Entries under the column “Mean Return” report the arithmetic average of geometric mean returns (in percentages) over the respective overlapping periods (numbers in parentheses are *t* statistics). The third and fourth column report results of the Davidson–Duclos (2000) test of stochastic dominance of winner over loser portfolios at second and third order respectively. DD test statistics are computed over a grid of 100 winner and loser portfolio returns. The table reports the percentage of DD statistics which are significant at the 5% significance level, based on the asymptotic critical value of 3.254 of the studentized maximum modulus (SMM) distribution. Bold entries indicate that winners dominate losers at the 5% significance level.

Holding period	Mean return		SSD		TSD	
	All mkts	Developed mkts	%DD<0	%DD>0	%DD<0	%DD>0
1-month	0.0357 (3.30)	0.0044 (0.62)	43	0	78	0
3-month	0.0040 (0.49)	0.0035 (0.70)	0	25	0	24
6-month	0.0348 (4.91)	0.0264 (8.03)	81	0	77	0
9-month	0.0241 (3.57)	0.0290 (8.80)	92	0	89	0
12-month	0.0053 (0.84)	0.0162 (5.53)	57	21	44	27

due to low liquidity, then we cannot claim that they are abnormal. To examine whether momentum profits are unique to emerging markets, we apply the momentum strategy to developed markets only by excluding the following markets from the sample: Indonesia, Korea, Malaysia, South Africa, Singapore, Spain, Taiwan and Thailand. This leaves us with 16 developed markets. For brevity, we report only the DD test results for this group of countries (Table 6).

The second column of Table 6 reports the average daily return of the momentum strategy for all 24 markets and for the developed markets only. Consistent with the liquidity story, there is weaker evidence of the momentum effect for developed markets than the overall sample. Nonetheless, the 6- and 9-month mean daily returns for developed markets are still quite large (0.0264% and 0.0290% respectively) and winners still dominate losers at the 1, 6 and 9-month horizon. Thus, the momentum effect is not unique to emerging markets. Our result is consistent with Griffin et al. (2003) who find that on average, a 6-month momentum strategy yields returns of 0.51% per month for developed markets and 0.27% for emerging markets.

8.3. Can momentum profits be explain by transaction costs?

Momentum strategies implemented on equally-weighted portfolios of stocks can incur non-trivial transaction costs in terms of direct brokerage costs, bid–ask spreads and price impact and these may negate gross momentum profits (see, e.g., [Lesmond et al., 2004](#)). The effect of transaction costs depends on the design of the momentum strategy (e.g., frequency of portfolio rebalancing, number and market size of securities in winner and loser portfolios, etc.). We mitigate transaction cost impact by focusing on value-weighted stock indices rather than individual stocks. More importantly, in contrast to previous momentum studies which rebalance portfolios monthly, we minimize the frequency of rebalancing to just twice for any holding period (i.e., at the start and end of each period). As such, transaction costs are unlikely to reverse the profitability of all momentum strategies. Nevertheless, to have a feel for the impact of transaction costs on momentum profits, we perform sensitivity tests using round trip trading costs of 2% and 3%. The benchmark study of [Jegadeesh and Titman \(1993\)](#) assumes a round-trip trading cost of 1%. This estimate is probably too low as it ignores bid–ask spreads and price impact. For international stock index funds, [Engle and Sarkar \(2002\)](#) estimate an average quoted bid–ask spread of 1.2% for seven MSCI Exchange Traded Funds, which includes one emerging market (Brazil) and 1.04% for developed markets. These spreads appear large, but they are actually quite close to the quoted spreads for the largest quintile of NYSE stocks (see [Lesmond et al., 2004](#)). Our 2% cost assumes a spread of 1.2% plus roundtrip commissions of 0.8% for transaction amounts of \$50,000 taken from [Lesmond et al. \(2004\)](#). The latter also provides indirect estimates of price impact costs, ranging from 11% to 23% of total trading cost depending on the momentum strategy. Our 3% total cost assumes a 1% price impact cost in addition to the aforementioned commissions and spreads.⁷

The results (available on request) indicate that these levels of transaction costs negate momentum profits for the 1, 3 and 12 month holding periods, but leave the profitability of the 6- and 9-month momentum strategies largely intact, especially in the second subperiod. At 2% transaction cost, the average annual net return for the 6- and 9-month momentum strategies in the second subperiod is 9.6% and 2.73%, respectively and at 3% transaction cost, the 6-month strategy is profitable (annual net return of 5.8%) but the 9-month strategy is not. These net returns are all

⁷ [Lesmond et al. \(1999\)](#) develop estimates of total trading costs that include implied effective spreads, commissions, price impact and immediacy costs. Our estimate of price impact cost is based on the momentum study of [Lesmond et al. \(2004\)](#) and assumes zero immediacy cost. From Table 2 of [Lesmond et al. \(2004\)](#), we estimate the average price impact cost as the difference between average total trading cost and the sum of mean quoted commissions and effective bid–ask spreads (direct and Roll effective spreads). Price impact cost defined this way amounts to between 11% and 23% of total trading cost based on the [Jegadeesh and Titman \(1993\)](#) momentum strategy (sample includes all NYSE/AMEX stocks) and between 12.4% and 13.2% for the [Jegadeesh and Titman \(2001\)](#) momentum study (which includes NASDAQ stocks but excludes stocks with share price under \$5 and stocks within the smallest size decile). As we assume a spread of 1.2% and 0.8% commission, our 3% total trading cost estimate implies a relatively steep price impact cost of one-third of total trading cost.

statistically significant. They are probably understated to the extent that some trades occur within quoted spreads (see Peterson and Fialkowski, 1994).

Momentum studies often underemphasize the point that momentum strategies can be profitable without having to short sell losers. Indeed, shorting past losers often *reduces* momentum profits, a fact that was documented as early as Jegadeesh and Titman (1993). Using 1965–1989 data, Jegadeesh and Titman (1993) report average semiannual returns of 10.44% obtained by longing past 6-month winners and a loss of 4.74% for shorting 6-month losers, implying that investors would have earned larger profits by a “long winner only” strategy. Lesmond et al. (2004) finds that this pattern has persisted. Using data from 1980–1998, they report a semiannual return of 10.35% from longing 6-month winners and a loss of 2.52% from shorting 6-month losers. After accounting for an estimated all-in transaction cost of 4.317%, the long-winner-only strategy still leaves investors with net returns of 6.03%. Korajczyk and Sadka (2004) find that as much as \$5 billion can be invested in a long-winners-only momentum strategy before excess returns net of transaction costs disappear. They conclude that trading costs in the form of spreads and price impacts cannot fully explain the momentum anomaly.

Our results are consistent with the above studies. From Table 2, the mean daily return from longing 6-month winners is 0.0521% for the full sample period or a semiannual return of about 6.51%. Deducting 2% transaction costs still leaves investors with a net return of 4.51%. Net returns for the two subperiods are 3.66% and 5.93% respectively, both statistically significant at 1%. Returns of these magnitudes cannot be easily explained away by transaction costs. Nor do transaction costs explain why continuation patterns exist in the first place. Furthermore, as Grundy and Martin (2001) point out, transaction costs are hardly relevant for a choice between two portfolios that differ only in their prior returns. The zero marginal costs of such a choice naturally induce a preference for past winners over past losers. This may well explain the persistence of momentum profits over the years.

9. Conclusions

Asset pricing models have difficulty accounting for the momentum phenomenon, which appears to exist globally. This paper eschews this standard approach by applying stochastic dominance criteria to test for the momentum effect. The main advantage of the stochastic dominance approach is that it makes minimal assumptions about the distribution of returns or investor risk preferences. Also, stochastic dominance rules consider the entire distribution of returns, not just the first two moments as in mean-variance analysis.

Preliminary analysis of the data shows that (a) the momentum effect exists globally, (b) the momentum strategy has remained profitable in recent times and (c) for most holding periods, momentum profits could have been earned by simply buying winners without having to short sell losers. Formal tests confirm that winners have stochastically dominated losers at second and third order in recent years. These

results are robust to our assumed level of transaction costs. Overall, these results indicate that the momentum effect remains an anomaly for the efficient market hypothesis and standard equilibrium asset pricing models that assume investor risk aversion.

Fans of efficient markets may argue that this conclusion is premature and that there must be some yet-to-be-discovered equilibrium asset pricing models capable of rationalizing the momentum phenomenon. It is clear from this study that such asset pricing models will either have to assume that investors are risk lovers or incorporate quite subtle aspects of investor's behavior toward risk.⁸ Theory aside, the testability of such sophisticated models will be a challenging issue for empiricists for years to come.

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⁸See, e.g., Barberis et al. (2001).

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